## Chemical degradation stoichiometric equations

The anaerobic degradation pathways that are used in LDAT are shown in Table 1. These are derived from those proposed by (Young 1995, Reichel et al. 2005, El-Fadel et al. 1996).

<table>
<thead>
<tr>
<th>Stage</th>
<th>Reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>First stage</td>
<td>Protein as ( \text{C}<em>{46}\text{H}</em>{77}\text{O}<em>{17}\text{N}</em>{12}\text{S} ) forms aqueous acid ( \text{C}<em>{4}\text{H}</em>{8}\text{O}<em>{2} ) and acetic acid ( \text{C}</em>{2}\text{H}<em>{4}\text{O}</em>{2} ) anaerobically ( \text{C}<em>{46}\text{H}</em>{76}\text{O}<em>{17}\text{N}</em>{12}\text{S} - 27.5\text{H}<em>{2}\text{O} = 7.39\text{C}</em>{4}\text{H}<em>{8}\text{O}</em>{2} + 5.15\text{C}<em>{2}\text{H}</em>{4}\text{O}<em>{2} + 6.14\text{H}</em>{2}\text{CO}_{3} + 12\text{NH}_3 + \text{H}_2\text{S} + \text{OH} )</td>
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<td>Fat represented as ( \text{C}<em>{56}\text{H}</em>{104}\text{O}<em>{6} ) forms aqueous and acetic acid anaerobically ( \text{C}</em>{56}\text{H}<em>{103}\text{O}</em>{6} + 9.88\text{H}<em>{2}\text{O} + 6.56\text{H}</em>{2}\text{CO}<em>{3} = 10.56\text{C}</em>{4}\text{H}<em>{8}\text{O}</em>{2} + 6.72\text{C}<em>{2}\text{H}</em>{4}\text{O}_{2} + 5.88\text{CH}_4 + \text{OH} )</td>
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<td>Carbohydrate high order forms aqueous acid anaerobically ( \text{C}<em>{12}\text{H}</em>{23}\text{O}<em>{12} + 2\text{H}</em>{2}\text{O} = 2\text{C}<em>{4}\text{H}</em>{8}\text{O}<em>{2} + \text{CH}<em>4 + 3\text{H}</em>{2}\text{CO}</em>{3} + \text{OH} )</td>
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<td></td>
<td>Glucose - Carbohydrate forms acetic acid anaerobically ( \text{C}<em>{6}\text{H}</em>{11}\text{O}<em>{6} + 2\text{H}</em>{2}\text{O} = 2\text{C}<em>{2}\text{H}</em>{4}\text{O}<em>{2} + \text{CH}<em>4 + \text{H}</em>{2}\text{CO}</em>{3} + \text{OH} )</td>
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<tr>
<td>Second stage</td>
<td>Aqueous acid ( \text{CH}_3(\text{CH}_2)_2\text{COOH} ), ( \text{C}_4\text{H}_8\text{O}_2 ) forms acetic acid anaerobically ( 4\text{C}_4\text{H}_8\text{O}<em>2 + 6\text{H}</em>{2}\text{O} = 4\text{C}_2\text{H}_4\text{O}_2 + 6\text{CH}<em>4 + 2\text{H}</em>{2}\text{CO}_3 )</td>
</tr>
<tr>
<td>Third stage</td>
<td>Acetic acid forms methane ( \text{C}_2\text{H}_4\text{O}_2 + \text{H}_2\text{O} = \text{CH}<em>4 + \text{H}</em>{2}\text{CO}_3 )</td>
</tr>
</tbody>
</table>

Table 1 Anaerobic degradation pathways

Associated with each of the primary degradation pathways is a bacteria growth and death pathway, (Barlaz et al. 1992).

Following (Reichel et al. 2005), for a general substrate \( \text{C}_a\text{H}_b\text{O}_c\text{N}_d\text{S}_e \) it is assumed that the growth pathways in which bacteria \( \text{C}_2\text{H}_7\text{NO}_2 \) are produced from the substrate are given by,

\[
5\text{C}_a\text{H}_b\text{O}_c\text{N}_d\text{S}_e + \text{ANH}_4^+ = a\text{C}_5\text{H}_7\text{NO}_2 + 4\alpha\text{DH}_2 + (1 - 4\alpha)\text{DH}^+ + \text{EH}_2\text{O} + 5\text{eSO}_4^- \quad (1)
\]

in which the hydrogen terms have been split to provide some flexibility in the \( \text{CH}_4 : \text{CO}_2 \) ratio that the model produces. This is based on the assumption that this is achieved by bacteria utilising hydrogen to convert \( \text{CO}_2 \) to \( \text{CH}_4 \) through,

\[
\text{CO}_2 + 4\text{H}_2 = \text{CH}_4 + 2\text{H}_2\text{O}
\]

Or, in terms of \( \text{H}_2\text{CO}_3 \)

\[
\text{H}_2\text{CO}_3 + 4\text{H}_2 = \text{CH}_4 + 3\text{H}_2\text{O}
\]

A stoichiometric balance of equation (1) gives,
\[(1 + 4\alpha)D = a + 5b - 10c - 20d + 40e\]
\[E = 5c - 2a - 20e\]
\[A = a - 5d\]

We can replace the term $4\alpha DH_2$ by $\alpha D(\text{CH}_4 - \text{H}_2\text{CO}_3) + 3\alpha DH_2 O$ which through the parameter $\alpha$ controls the extent to which $\text{CH}_4$ is produced at the expense of $\text{CO}_2$ ($\text{H}_2\text{CO}_3$) and which impacts on the $\text{CH}_4 : \text{CO}_2$ ratio.

Equation (1) then becomes,
\[5C_aH_bO_cN_dSe + \text{ANH}_4^+ = aC_aH_bNO_2 + aDCH_4 - aDH_2CO_3 + (1 - 4\alpha)DH^+ + (E + 3\alpha)H_2O + 5e\text{SO}_4^-
\]

(2)

With maximum conversion to $\text{CH}_4$ taking place when $\alpha = 0.25$.

The death pathway is, (Reichel et al. 2005),
\[6C_5H_7NO_2 + 19H_2O = 5\beta C_6H_{11}O_6^- + 6NH_4^+ + OH^- + 5(1 - \beta)I_{C_5H_7O_6^-} - 5(1 - \beta)H^+
\]

(3)

Where $C_6H_{11}O_6^-$ is degradable glucose and $I_{C_5H_7O_6^-}$ is an inert compound with mathematically the same molecular weight characteristics as glucose.

Here $\beta$ controls the tail developed as the result of the bacteria feeding back degradable glucose into the system as the populations die back. With $\beta = 0$ there is no feedback and the dead biomass forms an inert product. With $\beta = 1$ full feedback to degradable glucose is assumed to take place.

The influence on the $\text{CH}_4 : \text{CO}_2$ ratio of $\alpha$ and $\beta$ in equations (3) and (4) is however quite small because the bacteria pathways are thought to be only about 10% of the degradation chemistry the other 90% passing through the reactions given in Table 9, which have an output $\text{CH}_4 : \text{CO}_2$ ratio of around 50:50 by volume. Indeed some additional direct delivery of $\text{CH}_4$ is needed to add to equation (4) to achieve 60:40. Such an addition could take the form,
\[C_8H_7NO_2 + 2\text{H}^+ = 5\text{CH}_4 + \text{NH}_4^+ + 2\text{H}_2O\]

(4)

Theoretically a 60:40 ratio can be obtained by adding 20% of equation (4) into equation (3).

The aerobic pathways used in LDAT are shown in Table 2.
Aerobic degradation of Protein ion
\[ C_{46}H_{76}O_{17}N_{12}S - 49.75O_2 + 26.5H_2O = 46H_2CO_3 + 12NH_3 + SO_4^2- + OH^- \]

Aerobic degradation of Fat ion
\[ C_{55}H_{103}O_6 - 78O_2 + 4H_2O = 55H_2CO_3 + OH^- \]

Aerobic degradation of Carbohydrate ion
\[ C_{12}H_23O_6^2- + 12O_2 + H_2O = 12H_2CO_3 + OH^- \]

Aerobic degradation of Glucose ion
\[ C_6H_11O_6^- + 6O_2 + H_2O = 6H_2CO_3 + OH^- \]

Formation of biomass using protein \( C_{46}H_{76}O_{17}N_{12}S \)
\[ 5C_{46}H_{76}O_{17}N_{12}S - 14NH_4^+ = 46C_2H_2NO_2 + 56H^+ + 27H_2O + 5SO_4^{2-} \]

Formation of biomass using fat represented as \( C_{55}H_{103}O_6 \)
\[ 5C_{55}H_{103}O_6^- + 55NH_4^+ = 55C_2H_2NO_2 + 510H^+ - 80H_2O \]

Formation of biomass using carbohydrate high order
\[ 5C_{12}H_23O_6^2- + 12NH_4^+ = 12C_2H_2NO_2 + 7H^+ + 36H_2O \]

Formation of biomass using glucose (TUB)
\[ C_6H_11O_6^- + 1.2NH_4^+ = 1.2C_2H_2NO_2 + 0.2H^+ + 3.6H_2O \]

Nitrification by nitrosomonas bacteria
\[ NH_4^+ + 1.5O_2 = NO_2^- + 2H^+ + H_2O \]

Nitrification by nitrobactor bacteria
\[ NO_2^- + 0.5O_2 = NO_3^- \]

Formation of biomass following denitrification by nitrosomonas bacteria
\[ 5C_6H_11O_6^- + 6NH_4^+ = 6C_2H_2NO_2 + 18H_2O + H^+ \]

Formation of biomass following denitrification by nitrobactor bacteria
\[ 5C_6H_11O_6^- + 6NO_2^- + 18H_2 + 11H^+ = 6C_2H_2NO_2 + 30H_2O \]

Methane oxidation
\[ CH_4 + 2O_2 = H_2O + H_2CO_3 \]

Formation of biomass using Methane \( CH_4 \)
\[ 5CH_4 + NH_4^+ + 2H_2O = C_2H_2NO_2 + 21H^+ \]

Table 2 Aerobic pathways